Chordal Exercises

Practice

1. (a) Install the "Chordal" package. To do so, copy the file "Chordal.m2" and the folder "Chordal" into a new directory. Run M2 in this directory and execut `installPackage ''Chordal''`

   (b) Run the script "ex.m2". The option `JpgViewer` should be dropped if there are problems.

2. Find a chordal completion of the Peterson graph using the command `chordalGraph`. The Peterson graph can be obtained as `G = generalizedPetersonGraph(5,2)`. The command `displayGraph` shows the graph.

3. Let $I_{n,q}$ be the q-coloring ideal of the n-cycle graph. The function `colorIdealCycle(n,q)` of "ex.m2" computes $I_{n,q}$.

   (a) Verify that $I_{101,2}$ is infeasible using the commands `chordalElim(N); print N`.

   (b) Compute the elimination ideals of $I_{100,3}$.

4. Let I_n be the ideal of adjacent minors of a $2 \times n$ matrix. The function `adjMinorsIdeal` returns I_n and a chordal network representation.

   (a) Use the command `dim` to compute the dimension of I_n?

   (b) Use the command `codimCount` to determine the total number of components.

   (c) Use the command `nextChain` to get the first component of the network.

   (d) Use the command `displayNet` to visualize the chordal network.

5. Consider the ideal

$$I_{n,3} := \langle x_i x_j x_k : 1 \leq i < j < k \leq n \rangle \subset \mathbb{Q}[x_1, \ldots, x_n]$$

   Define a function that computes $I_{n,3}$ for any n (use Lex order). Find a chordal network representation of this ideal, using the commands `N = chordalNet I; chordalTria N; displayNet N`

6. Consider the 4-coloring problem explained by Mike (see file "Day 1/Mike/eg-4color.m2"). Solve the problem by computing the elimination ideals.

Theory

1. Show that the following definitions of chordal graphs are equivalent.

   (a) A graph is chordal if it has a perfect elimination ordering.

   (b) A graph is chordal if it has no induced cycle of length ≥ 4.

2. The *treewidth* is a very important parameter of a graph. The *clique number* of a graph is the size of the largest clique. The treewidth of a chordal graph is its clique number minus one. The treewidth of a (nonchordal) graph is the smallest treewidth among all possible chordal completions. What is the treewidth of: the path P_n? a tree? the cycle graph C_n? the complete graph K_n? the grid graph $P_n \times P_n$?